

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2010

MT 2501/ MT 2500 - ALGEBRA, ANAL.GEO & CALCULUS - II

Date & Time: 20/04/2010 / 1:00 - 4:00

Dept. No.

Max. : 100 Marks

PART – A

Answer ALL questions.

(10 × 2 = 20 marks)

1. Evaluate $\int \frac{dx}{(x+2)(x+1)}$.

2. If f is an odd function show that $\int_{-a}^a f(x)dx = 0$.

3. Solve $x\sqrt{1+y^2} + y\sqrt{1+x^2} \frac{dy}{dx} = 0$.

4. Solve $(D^2 + 4D + 4)y = 0$.

5. If $\sum_{i=1}^{\infty} u_i$ is convergent, show that $\lim_{n \rightarrow \infty} u_n = 0$.

6. State Cauchy's root test.

7. Find the coefficient of x^n in the expansion of $(3 + 2x)e^{-3x}$.

8. Write the expansion of $(1 - x)^{-2}$.

9. Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$.

10. Find the radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2z + 4y + 2z - 15 = 0$.

PART – B

Answer any FIVE questions

(5 × 8 = 40 marks)

11. Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$.

12. Solve $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 x}{y^2}$.

13. Solve $(D^2 + 2D + 5)y = x e^x$.

14. Test the convergence of $\sum_{n=0}^{\infty} \frac{n^3 + 1}{2^n + 1}$.

(P.T.O.)

15. Using Raabe's test, examine the convergence of $\frac{1^2}{2^2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2} + \dots$

16. For sufficiently large values of x , show that $\sqrt{x^2 + 16} - \sqrt{x^2 - 9} = \frac{7}{2x}$ approximately.

17. Sum the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots$

18. Find the image of the point $(1, -2, 3)$ in the plane $2x - 3y + 2z + 3 = 0$.

PART - C

Answer any TWO questions

(2 × 20 = 40 marks)

19. (a) If $I_{m,n} = \int \sin^m x \cos^n x dx$ (m, n being positive integers), show that $(m+n)I_{m,n} = \cos^{n-1} x + (n-1)I_{m,n-2}$ and hence evaluate $\int \sin^6 x \cos^3 x dx$.

(b) Find the length of one loop of the curve $3ay^2 = x(x-a)^2$.

20. (a) Solve: $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$.

(b) Examine the convergence of $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{1/2} x^n$.

21. (a) Sum the series to infinity $\frac{1 \cdot 4}{5 \cdot 10} + \frac{1 \cdot 4 \cdot 7}{5 \cdot 10 \cdot 15} + \dots$.

(b) Sum the series to infinity $\frac{5}{1 \cdot 2 \cdot 3} + \frac{7}{3 \cdot 4 \cdot 5} + \frac{9}{5 \cdot 6 \cdot 7} + \dots$.

22. (a) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$; $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$ and the equations of the Line of shortest distance.

(b) Find the equation of the sphere through the four points $(2, 3, 1)$, $(5, -1, 2)$, $(4, 3, -1)$ and $(2, 5, 3)$.

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